

Research article

# **DISPERSION INFLUENCES TO PREDICT AMMONIA AND POTASSIUM DEPOSITION PRESSURED BY HOMOGENOUS POROSITY AND VELOCITY IN SHORT FRESH WATER AQUIFER IN COASTAL AREA OF BAKANA, RIVERS STATE OF NIGERIA.**

**Eluozo, S. N.**

Subaka Nigeria Limited, Port Harcourt, Rivers State of Nigeria  
Director and Principal Consultant, Civil and Environmental Engineering,  
Research and Development  
E-mail: Soloeluzo2013@hotmail.com  
E-mail: [solomoneluzo2000@yahoo.com](mailto:solomoneluzo2000@yahoo.com)

---

## **Abstract**

Dispersion influences from deposited ammonia and potassium were examined in coastal area of Bakana, the deposition of shallow depth in the study location are pressured by deltaic nature of the formation, these include environmental condition and geological setting of the study location, the Phreatic zone has lots of challenges due to all these depositional rate of the substance, exponential rate of migration were experiences from the investigation that subjected modeling of ammonia and potassium transport, the rate of deposition in such coastal environment was as a result of homogeneous predominant deposition of velocity and concentration of the substances, porosity in uniform setting express it pressures through it migration to Phreatic zone within a short period, the derived solution express several method to evaluate various function of every parameters that formulated the system, these application generated the derived model that will predict ammonia and potassium level in coastal fresh aquiferous zone of Bakana.

**Copyright © WJWRES, all rights reserved.**

**Keyword:** dispersion influences, Ammonia, potassium porosity, velocity and fresh water aquifers

---

## 1. Introduction

A major current scientific challenge is scaling from the functional properties of organisms to processes at the ecosystem and global levels (Enquist et al. 2003; Torsvik and Ovreas 2002; Zak et al. 2006). Microbial respiration is a process that has particular importance in the ecosystem and global scales, representing about half of total CO<sub>2</sub> flux from soils (Hanson et al. 2000). Furthermore, effects of human-induced climate change on soil microbial communities and their metabolic activities could create potentially devastating feedbacks to the Earth's biosphere (Meir et al. 2006). Biomass made up of fast-growing species respire faster than an equal amount of biomass made up of slow-growing species. Microbes with low growth yields (biomass produced per unit substrate consumed) convert a larger fraction of substrate into CO<sub>2</sub> during growth, and so respire faster than efficiently growing organisms. It has been observed that there is an inevitable thermodynamic trade-off between growth rate and yield among heterotrophic organisms (Pfeiffer et al. 2001, Eluozo and Nwaoburu 2013). Past authors have proposed that two opposing ecological strategies exist at either end of this spectrum: a fast-growing, low yield competitive strategy and a slow growing high yield cooperative strategy (Kreft and Bonhoeffer 2005; Pfeiffer et al. 2001). For microbes, the cooperative, slow, efficient growth strategy is more successful in spatially structured environments such as biofilms (Kreft 2004; Kreft and Bonhoeffer 2005; MacLean and Gudelj 2006; Pfeiffer et al. 2001). With over a billion individual cells and estimates of 10<sup>4</sup>–10<sup>5</sup> distinct genomes per gram of soil (Gans et al., 2005; Tringe et al., 2005; Fierer et al., 2007b, David et al 2008), bacteria in soil are the reservoirs for much of Earth's genetic biodiversity. This vast phylogenetic and functional diversity can be attributed in part to the dynamic physical and chemical heterogeneity of soil, which results in spatial and temporal separation of microorganisms (Papke and Ward, 2004 Katherineel al 2011 Eluozo and Nwaoburu 2013). Given the high diversity of carbon (C) – rich compounds in soils, the ability of each taxon to compete for only a subset of resources could also contribute to the high diversity of bacteria in soils through resource partitioning (Zhou et al., 2002). Indeed, Waldrop and Firestone (2004) have demonstrated distinct substrate preferences by broad microbial groups in grassland soils and C resource partitioning has been demonstrated to be a key contributor to patterns of bacterial co-existence in model communities on plant surfaces (Wilson and Lindow, 1994 Eluozo and Nwaoburu 2013).

## 2. Theoretical background

Short fresh water aquifer in coastal environment has been investigated to have several challenges in deltaic formation, the deposition of shallow aquiferous zone in Phreatic condition are expressed through several environmental influences, these condition are peculiar in deltaic deposition, the coastal environments are made of mangrove forest, such condition through its climatic condition express various influences from high rain intensities including geological depositional pressures, the deposition of short fresh water aquifer cannot be completed without the deposition of natural and manmade minerals in the formation, these influences the deposition of short fresh water aquifers in the study area, the predominant deposition in coastal location is saline through the alluvia deposition of homogeneous stratification in deltaic formation, for simplicity, constant rain fail in those coastal environment infiltrate water in the soil, degree of saturation increase within various intercedes of the strata to ground water level, the same condition infiltrate pollutant such as ammonia and potassium to Phreatic zone, the deposition

of shallow short fresh water aquifer are pressured by lots of these geological deposited variations, base on these condition shallow Phreatic deposition experiences accumulated substances in coastal formation generating lots of substances migration within a short period, these conditions were experiences in coastal area of Bakana, most settlers may not take notices of it due to lack of orientation on quality ground water and its utilizations, the study is imperative because the deposition of shallow fresh water qualities has not been thoroughly examined to determine it rate of quality and its application for various purpose, exploration of short fresh ground water aquifers are normal ground water abstraction in coastal environment of Bakana, the rate of aquiferous heterogeneous deposition has not been examined in the study area, these has generated lots of different pollution challenges increasing hundreds of people illness in the study area , these research work if applied will monitor the rate of deposition including prevention of the contaminant in the study area.

### 3. Governing Equation

$$D_L \frac{\partial C}{\partial t} = \overline{\Phi V} \frac{\partial^2 C}{\partial Z^2} - \frac{\partial C}{\partial Z} \dots\dots\dots (1)$$

The behaviour of the substance is expressed through the developed governing equation, lots of depositional variations of the contaminant are found through the investigation carried out in the study location. Such reaction are found on physical process from the litho units of the formation, the coastal environment develop sub surface complexity under the influences of the predominant deposition tidal and saline , other environmental influences played some roles in the study location, but the major parameters were integrated to formulate a system producing the governing equation.

Boundary condition  $C(0,t) = C_0$  for  $t > 0$  ( $z, 0$ ) and  $(\infty,t) = C_0$  for  $t \geq 0$

The Laplace transform for a function  $f(t)$  which is defined for all values of  $t \geq 0$  is given.

$$\rho f(z) = f(s) = \int_0^{\infty} e^{-sz} f(z) dz \quad f(z) = \rho^{-1} f(s) \dots\dots\dots (2)$$

$$\rho f(z) = s\rho = s\rho f(z) - f(0) \text{ where } \rho^1(z) = \frac{\partial f}{\partial Z} \dots\dots\dots (3)$$

Taking the Laplace transform of the function c with respect to t eqn (1) changes to

$$D_L \rho \left[ \frac{\partial C}{\partial t} \right] = \overline{\Phi V} \left[ \frac{\partial^2 C}{\partial Z^2} \right] - \rho \frac{\partial C}{\partial Z} \dots\dots\dots (4)$$

Where  $D_L \rho \left[ \frac{\partial C}{\partial Z} \right] = D_L \rho(c) - C(z, 0)$

[C is a function of z and t i.e.  $C(z, t) = f(t)$ , therefore  $\rho f(t) = \rho C(z, t) = \bar{C}$ ]

$$\text{Let } \bar{C} = D_L \rho(c) \text{ then } \rho \left[ \frac{\partial C}{\partial Z} \right] = \frac{\partial}{\partial Z} \rho(C) = \frac{\partial \bar{C}}{\partial Z} \text{ and } \rho \left[ \frac{\partial^2}{\partial Z^2} \right] = \frac{\partial^2}{\partial Z^2} \rho(c) = \frac{\partial^2 \bar{C}}{\partial Z^2}$$

Where  $\bar{C}(z) = \rho C(z, t)$ , that is only t changes to s and z is unaffected and s is the Laplace parameter.

$$\text{At } z = 0: \bar{C}(z) = \int_0^{\infty} e^{-st} C(z, t) dt = \int_0^{\infty} e^{-st} C_o dt = \frac{\infty}{0} \left| -\frac{1}{s} e^{-st} C_o \right| = \frac{C_o}{s}$$

$$\text{At } z = \infty: \bar{C}(z) = \int_0^{\infty} e^{-st} C(z, t) dt = 0$$

Therefore at  $z = 0$ ,  $\bar{C}(z) = \frac{C_o}{s}$ , and at  $z = \infty$ ,  $\bar{C}(z) = 0$

[Since this is one dimensional flow equation, partial derivative changes to the full derivative, s is a Laplace parameter, which disappears on taking the inverse].

From the substitution Eqn

$$D_L s \bar{C} = \overline{\Phi V} \left[ \frac{d\bar{c}}{\alpha z} \right] - \left[ \frac{d\bar{c}}{dz} \right] \dots \dots \dots (5)$$

Let  $\bar{C} = A e^{\lambda z}$  be the solution of the above linear ordinary differential equation. [This is a standard way of solving this class of equations].

The expression here display homogeneous setting where the formation stratification deposit homogeneous strata as expressed in the study location, such condition were derived applying such equation, it will express the homogeneous condition of the concentration within some region of the formation that experiences homogeneous velocity of solute transport in the coastal environment.

$$\text{The } \frac{d\bar{c}}{dz} = A \lambda e^{\lambda z} \text{ and } \frac{d^2 \bar{c}}{dz^2} = A \lambda^2 e^{\lambda z} \dots \dots \dots (6)$$

Solution of these values in Eqn (5) gives

$$D_L A \lambda^2 e^{\lambda z} = \overline{\Phi V} A \lambda e^{\lambda z} - \phi \lambda e^{\lambda z} \text{ or } \left[ e^{\lambda z} = \lambda^2 \frac{\overline{\Phi V}}{D_L} \lambda - \frac{s}{D_L} \right] \dots \dots \dots (7)$$

This will be a solution of the auxiliary equation or the characteristics Equation = 0, this implies that

$$\left[ \lambda^2 - \frac{\overline{\Phi V}}{D_L} \lambda - \frac{s}{D_L} \right] = 0 \quad \dots\dots\dots (8)$$

Equation (8) is the standard quadratic equation and the solution is expressed in this form.

$$\lambda = \frac{\frac{\overline{\Phi V}}{D_L} \pm \sqrt{\frac{\overline{\Phi V}^2}{D_L^2} + \frac{4s}{D_L}}}{2}$$

That is  $\lambda_1 = \frac{\overline{\Phi V} + \sqrt{\overline{\Phi V}^2 + 4sD_L}}{2D_L}$  and  $\lambda_2 = \frac{\overline{\Phi V} - \sqrt{\overline{\Phi V}^2 + 4sD_L}}{2D_L}$

Therefore, either  $\overline{C} = Ae^{\lambda_1 z}$  or  $\overline{C} = Ae^{\lambda_2 z}$  is a solution. However, only the latter satisfies the boundary condition.

At  $z = \infty$ ,  $\overline{C} = \frac{C_o}{s}$ ,  $e^{-\infty} = 0$  {because  $\lambda_2$  is -ve and  $\lambda_1$  is +ve}

Therefore  $\overline{C} = A \left[ e^{\frac{\overline{\Phi V} - \sqrt{\overline{\Phi V}^2 + 4sD_L}}{2D_L} z} \right]$  is the solution

At  $Z = 0$   $\overline{C} = \frac{C_o}{s}$  give  $A = \frac{C_o}{s}$

Therefore  $\overline{C} = \frac{C_o}{s} \left[ \exp \left[ \exp^{\frac{\overline{\Phi V} - \sqrt{\overline{\Phi V}^2 + 4sD_L}}{2D_L}} \right] \right]$  is the solution  $\dots\dots\dots (9)$

From Equation (9)  $C(z,t)$  can be determined as  $\rho^{-1} \overline{C}(z)$

Equation (9) can further be expressed as:

$$C_o \exp \left( \frac{\overline{\Phi V} z}{2D_L} \right) - \frac{1}{\phi s} \exp \left[ \frac{-z}{\sqrt{D_L}} \left( \frac{\overline{\Phi V}^2}{4D_L} + s \right)^{\frac{1}{2}} \right]$$

Similar conditions are experienced in the derived solution in [9], these are parameters that establish their various function in their depositional reaction with ground water at different strata of phreatic zone. The derived solution express these condition through quadratic application, these condition implies that the predominant formation characteristics such as high degree of porosity may pressure the exponential phase of the contaminant, these application become use for such

Phase of the transport including depositional level of the substances

Application of the inverse Laplace transform to the above equation gives

$$C(z,t) = \rho^{-1} \bar{C}(z) = \rho^{-1} \left[ C_o \exp\left(\frac{\Phi \bar{V} z}{2D_L}\right) - \frac{1}{s} \exp\left[\frac{-z}{\sqrt{D_L}} \left(\frac{\Phi \bar{V}^2}{4D_L} + s\right)^{\frac{1}{2}}\right] \right]$$

$$= C(z,t) = \rho^{-1} \bar{C}(z) = \rho^{-1} \left[ C_o \exp\left(\frac{\Phi \bar{V} z}{2D_L}\right) \rho^{-1} \left[ \frac{1}{s} \exp\left[\frac{-z}{\sqrt{D_L}} \left(\frac{\Phi \bar{V}^2}{4D_L} + s\right)^{\frac{1}{2}}\right] \right] \right] \dots \dots \dots (10)$$

From the Laplace transform table

$$\rho^{-1} \left( \frac{1}{s} \exp\left(-\alpha \sqrt{\beta^2 + s}\right) \right) = \int_0^t \frac{\alpha}{2\sqrt{\pi} \beta} \exp\left[-\left(\frac{\alpha^2}{4u} + \beta^2 u\right) du\right] \dots \dots \dots (11)$$

Here  $\frac{Z}{\sqrt{D_L}}$  and  $\beta = \frac{\phi \bar{V}}{2\sqrt{D_L}}$

Therefore

$$C(z,t) = \rho^{-1} \bar{C}(z) = C_o \exp\left(\frac{\Phi \bar{V} z}{2D_L}\right) \left[ e^{-\alpha \beta} \int_0^t \frac{\alpha}{2\sqrt{\pi} \beta} \exp\left[-\frac{\alpha^2}{4u} - \beta^2 u + \alpha \beta du\right] \right] \dots \dots \dots (12)$$

The term in the bracket =  $\left[ e^{-\alpha \beta} \int_0^t \frac{\alpha}{2\sqrt{\pi} \beta} \exp\left[\frac{(\alpha - 2\beta u)^2}{4u} du\right] \right] \dots \dots \dots (13)$

$$= e^{-\alpha\beta} \int_0^t \left[ \frac{\alpha + 2\beta u}{4\sqrt{\pi u^3}} + \frac{\alpha - 2\beta u}{4\sqrt{\pi u^3}} \right] \exp \left[ -\frac{(\alpha - 2\beta u)^2}{4u} du \right] \dots\dots\dots (14)$$

$$= e^{-\alpha\beta} \left[ \int_0^t \frac{\alpha + 2\beta u}{4\sqrt{\pi u^3}} \exp \left[ \frac{(\alpha - 2\beta u)^2}{4u} du \right] + e^{2\alpha\beta} \int_0^t \frac{\alpha - 2\beta u}{4\sqrt{\pi u^3}} \exp \left[ \frac{(\alpha + 2\beta u)^2}{4\sqrt{\pi u^3}} du \right] \right] \dots\dots\dots (15)$$

Let  $\frac{\alpha - 2\beta u}{\sqrt{4u}} = A$  and  $\frac{\alpha + 2\beta u}{\sqrt{4u}} = B$  ..... (16)

Differentiating the term in Equation (16) give

$$\frac{dA}{du} = \frac{\sqrt{4u}(0 - 2\beta) - 2 \frac{1}{2} \frac{1}{\sqrt{2}} \sqrt{u} (\alpha - 2\beta u)}{4u} \quad \text{and} \quad \frac{\sqrt{4u}(0 + 2\beta) - 2 \frac{1}{2} \frac{1}{\sqrt{2}} \sqrt{u} (\alpha + 2\beta u)}{4u} \dots\dots\dots (17)$$

$$\text{Or } \frac{dA}{du} = \frac{-4\beta\sqrt{u} - \frac{\alpha}{u} + 2\beta\sqrt{u}}{4u} = \frac{-2\beta u - d}{4\sqrt{u^3}} = \frac{-(\alpha + 2\beta u)}{4\sqrt{u^3}}$$

$$\text{And } \frac{dB}{du} = \frac{4\beta\sqrt{u} - \frac{\alpha}{u} - 2\beta\sqrt{u}}{4u} = \frac{2\beta u - d}{4\sqrt{u^3}} = \frac{-(\alpha - 2\beta u)}{4\sqrt{u^3}}$$

$$\text{Or } dA = \frac{-(\alpha + 2\beta u)}{4\sqrt{u^3}} du \quad \text{and} \quad dB = \frac{-(\alpha - 2\beta u)}{4\sqrt{u^3}} du \dots\dots\dots (18)$$

$$C(z, t) = C_o \exp \left( \frac{\Phi V z}{2D_L} \right) \left[ - \int_0^{\frac{\alpha - 2\beta t}{\sqrt{4t}}} \exp(-A^2) \frac{dA}{\sqrt{\pi}} - e^{2\alpha\beta} \int_{\infty}^{\frac{\alpha + 2\beta t}{\sqrt{4t}}} \exp(-\beta^2) \frac{dB}{\sqrt{\pi}} \right] \dots\dots\dots (19)$$

For the limit when  $u = 0$

$$A = \frac{\alpha - 2\beta \cdot 0}{0} = \infty \quad B = \frac{\alpha + 2\beta \cdot 0}{0} = \infty, \text{ and when}$$

$$u = t, A = \frac{\alpha - 2\beta t}{\sqrt{4t}} \quad \text{and} \quad B = \frac{\alpha + 2\beta : t}{\sqrt{4t}} du$$

Changing the integral limits in Equation (19), it is given as

$$\frac{1}{2} \frac{2}{\sqrt{\pi}} e^{-\alpha\beta} \int_{\frac{\alpha-2\beta t}{\sqrt{4t}}}^{\infty} \exp(-A^2) dA + \frac{1}{2} \frac{2}{\sqrt{\pi}} e^{\alpha\beta} \int_{\frac{\alpha-2\beta t}{\sqrt{4t}}}^{\infty} \exp(-B^2) dB \quad \dots\dots\dots (20)$$

The complimentary error function is defined as  $erfc x = \frac{2}{\sqrt{\pi}} \int_{\frac{\alpha-2\beta t}{\sqrt{4t}}}^{\infty} \exp(-t^2) dt$

For which Equation (20) changes to

$$\frac{e^{-\alpha\beta}}{2} erfc \frac{\alpha-2\beta t}{\sqrt{4t}} + \frac{e^{-\alpha\beta}}{2} erfc \frac{\alpha+2\beta t}{\sqrt{4t}} \quad \dots\dots\dots (21)$$

The various combinations of  $\alpha$  and  $\beta$  can be simplified as follows:

$$\alpha\beta = \frac{Z}{D_L} \frac{\overline{\Phi V}}{2\sqrt{D_L}} = \frac{\overline{\Phi V z}}{2\sqrt{D_L}}; \quad \frac{\alpha+2\beta t}{\sqrt{4t}} = \frac{\frac{Z}{\sqrt{D_L}} + \frac{\overline{\Phi V t}}{\sqrt{D_L}}}{2\sqrt{t}} = \frac{Z + \overline{\Phi V t}}{2\sqrt{D_L t}} \text{ And}$$

$$\frac{\alpha-2\beta t}{\sqrt{4t}} = \frac{\frac{Z}{\sqrt{D_L}} + \frac{\overline{\Phi V t}}{\sqrt{D_L}}}{2\sqrt{t}} = \frac{Z - \overline{\Phi V t}}{2\sqrt{D_L t}}$$

Using these, equation (21) changes to

$$e \frac{\overline{\Phi V Z}}{2D_L} erfc \left[ \frac{Z - \overline{\Phi V t}}{2\sqrt{D_L t}} \right] + e \frac{\overline{\Phi V Z}}{2D_L} erfc \left[ \frac{Z + \overline{\Phi V t}}{2\sqrt{D_L t}} \right]$$

Therefore finally, Equation (11) with equation (14) changes to

$$C(z,t) = C_o \exp \left( \frac{\overline{\Phi V z}}{2D_L} \right) \frac{1}{2} \exp \left( -\frac{\overline{\Phi V z}}{2D_L} \right) erfc \left[ \frac{Z - \overline{\Phi V t}}{2\sqrt{D_L t}} \right] \frac{1}{2} \exp \left( \frac{\overline{\Phi V z}}{2D_L} \right) erfc \left[ \frac{Z + \overline{\Phi V t}}{2\sqrt{D_L t}} \right]$$

Or  $C(z,t) = \frac{C_o}{2} \left[ erfc \left[ \frac{Z - \overline{\Phi V t}}{2\sqrt{D_L t}} \right] + \exp \left( \frac{\overline{\Phi V z}}{D_L} \right) erfc \left[ \frac{Z + \overline{\Phi V t}}{2\sqrt{D_L t}} \right] \right] \quad \dots\dots\dots (22)$

Declining of the these substances may be experiences in short fresh water aquifers , but before such condition take place , since there lots of variation in the coastal environment, the influences of these variation must be noted on their various roles in the depositional level of the contaminants, the development of the governing equation were express through the various derived process considering these variation in the environment, these process were expressed base on the behaviour of the substances under the influences of the deltaic deposition of the formation, the derive solution establish several conditions base on these phase of the system, the substances in some region



express degradation predominant in Phreatic zone in the coastal environment, base on these condition errors functions were applied to produce final equation for the study.

#### 4. Conclusion

The deposition of short fresh water aquifer in the coastal environment has been thoroughly expressed, these are base on the behaviour of the substance deposited in the study location, the behaviour of the formation were expressed thus the influences on the substances were evaluated, the study were able to determined the challenges in short fresh water quality through the deposition of the substances, the study of ammonia and potassium in study area are very important because of high deposition of microbes of different species, because they are substrate utilization to microbial growth, the situation has generated lots of challenges that must be address if quality water will be abstracted in the study location. The development of these model has express lots of consequences on the deposition of ammonia and potassium in the deltaic formation, experts will definitely make their monitoring and evaluation easy if they apply these developed model to monitor the rate of deposition that can also predict the growth rate of other deposited microbes in coastal short fresh aquifer in the study environment.

#### References

- [1] Gans, J., Wolinsky, M., and Dunbar, J. (2005). Computational improvements reveal great bacterial diversity and high metal toxicity in soil. *Science* 309, 1387–1390.
- [2] Zhou, J., Xia, B., Treves, D. S., Wu, L. Y., Marsh, T. L., O'Neill, R. V., Palumbo, A. V., and Tiedje, J. M. (2002). Spatial and resource factors influencing high microbial diversity in soil. *Appl. Environ. Microbiol.* 68, 326–334.
- [3] Tringe, S. G., Von Mering, C., Kobayashi, A., Salamov, A. A., Chen, K., Chang, H. W., Podar, M., Short, J. M., Mathur, E. J., Detter, J. C., Bork, P., Hugenholtz, P., and Rubin, E. M. (2005). Comparative metagenomics of microbial communities. *Science* 308, 554–557
- [4] Fierer, N., Bradford, M. A., and Jackson, R. B. (2007a). Toward and ecological classification of soil bacteria. *Ecology* 88, 1354–1364.
- [5] Papke, R. T., and Ward, D. M. (2004). The importance of physical isolation to microbial diversification. *FEMS Microbiol. Ecol.* 48, 293–303
- [6] Papke, R. T., and Ward, D. M. (2004). The importance of physical isolation to microbial diversification. *FEMS Microbiol. Ecol.* 48, 293–303
- [7] Wilson, M., and Lindow, S. E. (1994). Coexistence among epiphytic bacterial populations mediated through nutritional resource partitioning. *Appl. Environ. Microbiol.* 60, 4468–4477.
- [8] Enquist BJ, Economo EP, Huxman TE, Allen AP, Ignace DD, Gillooly JF (2003) Scaling metabolism from organisms to ecosystems. *Nature* 423:639–642. doi:10.1038/nature 01671

[9] Hanson PJ, Edwards NT, Garten CT, Andrews JA (2000) Separating root and soil microbial contributions to soil respiration: a review of methods and observations. *Biogeochemistry* 48:115–146. doi:10.1023/A:1006244819642

[10] Meir P, Cox P, Grace J (2006) The influence of terrestrial ecosystems on climate. *Trends Ecol Evol* 21:254–260. doi:10.1016/j.tree.2006.03.005

[11] Pfeiffer T, Bonhoeffer S (2004) Evolution of cross-feeding in microbial populations. *Am Nat* 163:E126–E135. doi: 10.1086/383593

[12] Pfeiffer T, Schuster S, Bonhoeffer S (2001) Cooperation and competition in the evolution of ATP-producing pathways. *Science* 292:504–507. doi:10.1126/science.1058079

[13] Torsvik V, Ovreas L (2002) Microbial diversity and function in soil: from genes to ecosystems. *Curr Opin Microbiol* 5:240–245. doi:10.1016/S1369-5274(02)00324-7

[14] Kreft JU (2004) Biofilms promote altruism. *Microbiology* 150:2751–2760. doi:10.1099/mic.0.26829-0

[15] Kreft JU, Bonhoeffer S (2005) the evolution of groups of cooperating bacteria and the growth rate versus yield trade-off. *Microbiology* 151:637–641. doi:10.1099/mic.0. 27415-0

[16] David A. Lipson E Russel K. Monson Æ Steven K. Schmidt E Michael N. Weintraub 2008; The trade-off between growth rate and yield in microbial communities and the consequences for under-snow soil respiration in a high elevation coniferous forest *Biochemistry Springer Science+Business Media*

[17] Katherine C. Goldfarb<sup>1†</sup>, Ulas Karaoz<sup>1</sup>, China A. Hanson<sup>2</sup>, Clark A. Santee<sup>1</sup>, Mark A. Bradford<sup>3</sup>, Kathleen K. Treseder<sup>2</sup>, Matthew D. Wallenstein<sup>4</sup> and Eoin L. Brodie<sup>1</sup> 2011 Differential growth responses of soil bacterial taxa to carbon substrates of varying chemical recalcitrance *frontier in microbiology original research article*

[18] Eluozo, S. N. Nwaoburu A .O (2013) Mathematical model to monitor the deposition of void ratio and dispersion of phosphorus influence in Salmonella growth rate in coarse and gravel formation in Borikiri, rivers state of Nigeria *American Journal of Engineering Science and Technology Research* Vol. 1, No. 4, PP: 59 - 67,